Project proposal: Inferential statistics of rigid body stress states

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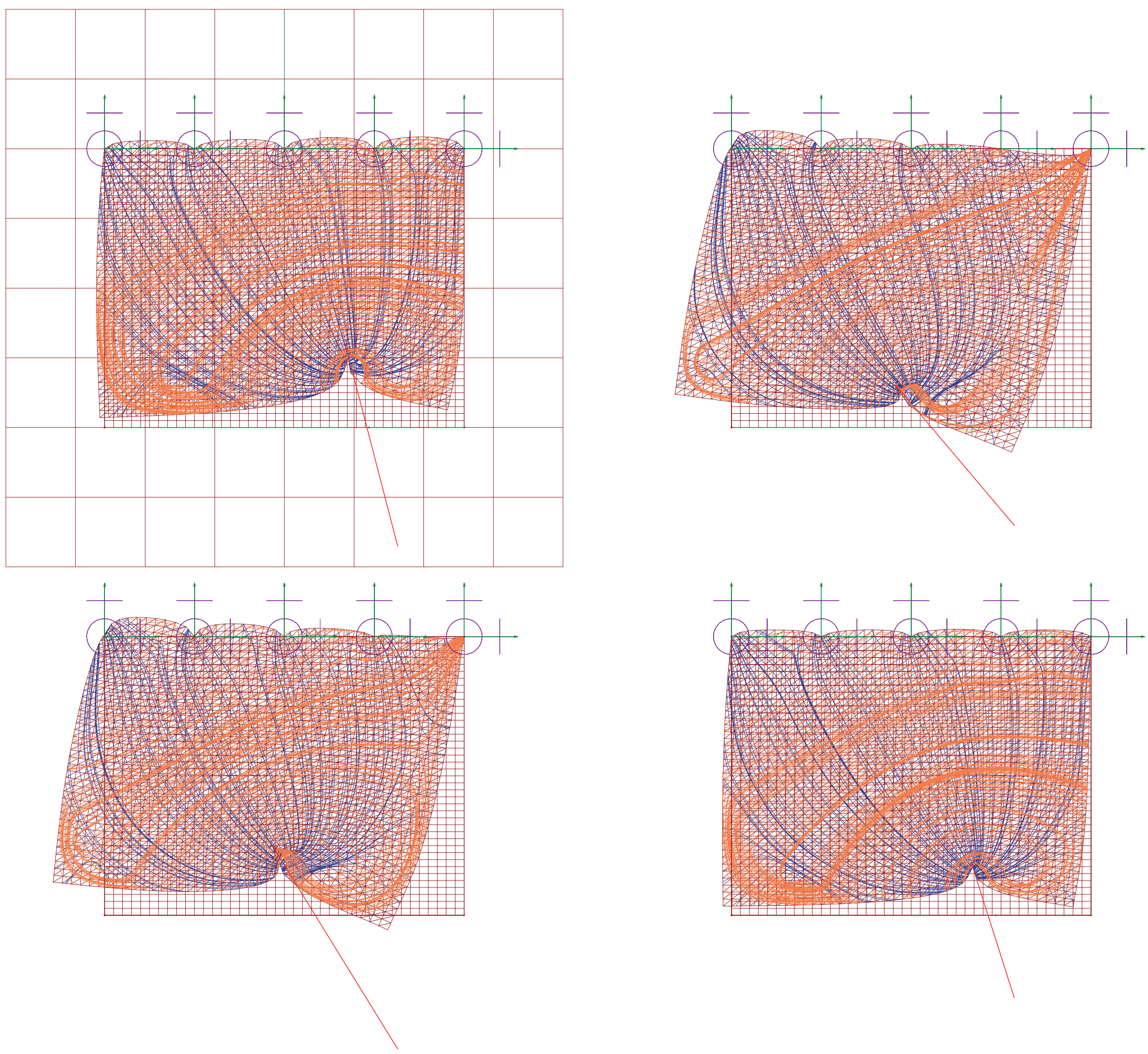
The goal of this structural mechanical project is to apply a new method for the analysis of large numbers of measurements and/or finite element computations of non-isotropic compressive or tensile stress states of a rigid body in continuum mechanics.
The problem we face is that of a lack of gold-standard statistical analysis procedure for symmetric positive-definite tensors such as the Cauchy stress tensor.
We thus propose a procedure allowing the researcher/mechanic to compare two given samples of stress states and assigning quantitative measure of certainty about the degree of empirical confidence about their equality of means. The stress samples can be obtained either from a finite element simulation, or from real measurements, or both.

Figure: Series of four finite element calculations of a 2d rectangular element under shear stress, subject to random perturbations in each iteration, with first and second principal stress lines in blue and orange (deformations not to scale).
In this project, we develop a framework for statistical analysis of stress fluctuations as in this series. In each subfigure, the point at the centre is associated with a particular two-by-two Cauchy stress tensor. A component-wise averaging procedure, resulting in the two-by-two mean Cauchy stress is not an adequate averaging procedure because it disrespects the magnitudes of its eigenvalues, the principal stresses. However, we propose a methodology for averaging and comparing large numbers of perturbed stress samples, circumventing this problem. We thereby allow the researcher to compute measures for the degree of certainty of difference in means between two sets of samples. An important application is quality control of finite element simulations.



For instance, an application scenario is the following. If one large sample of measurements comes from a finite element simulation and another sample comes from actual material measurements, the method can be used to answer the question whether the simulation reflects the actual distribution of stress tensors accurately. Thus, we can compute quantitative simulation quality measures. Obtaining these is a problem that has not yet been solved satisfactorily in the literature.

As another application, the method allows to compare large sets of measurements in two different materials and to answer the question whether there is sufficient statistical evidence that the stress state has changed between the materials regardless of measurement uncertainty.

There are numerous other possible application scenarios, for instance to form parametrization and form finding.

The mathematical underpinnings of the new method are roughly as follows. In classical statistics of scalar quantities, the quality of a large sample of observations is expressed by measures such as mean and variance, allowing, for instance, for comparison of two different samples and assigning a quantitative measure of significance to observed differences in means between two sets of samples.

In the case of stress states, however, the mean is not a robust summary statistic, nor is there any immediate stand-in candidate for the variance.

One reason for this, for example, is that the magnitudes of the principal stress computed from averaged components of the stress tensor, can differ strikingly from the averaged principal stresses of each single measurement. This effect leads to a high unreliability of averaging stress tensors in a component-wise, which is a problem becuase the main quantities affecting material state are the magnitudes of the principal stresses.

Here, we propose a linearization procedure for stress tensors coming from unitary representation theory, a classical branch of Pure Mathematics that has found, so far, few if any applications in structural mechanics. The linearization consists in lifting appropriately from the set of Cauchy tensors reflecting and then applying unitary representations as a means of linearizations. Computing mean and variance makes perfect sense in each unitary representation, thereby solving the problem explained above.

It is possible to extend the project's concept from elasticity to plasticity, by applying it to a suitable measurement of Piola-Kirchhoff tensors.

I have a PhD in Pure Mathematics, and have extensive experience in statistics, machine learning, as well as in Architecture. I have peer-reviewed publications in Pure Mathematics, statistics, machine learning as well as in structural mechanics.

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